

A Singlet-pairing superconductor is always also a super-spin-current-conductor

Chia-Ren Hu*

Department of Physics, Texas A&M University, College Station, TX 77843-4242, USA

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It is shown that, in a conductor carrying a moderate spin current, singlet pairing can still take place almost as effectively as without a spin current. The system will still be fully gapped for all spin-up and -down single-particle excitations, with no depaired electrons. All thermodynamic properties will be practically the same as without the spin current. All universality relations and laws of corresponding states found in the theory of Bardeen, Cooper, and Schrieffer should remain valid. Thus a singlet-pairing superconductor can always carry a persistent and dissipation-less spin-current. A heuristic argument to support this conclusion is also given, as well as two possible experimental tests of this prediction.

The Bardeen, Cooper, Schrieffer (BCS) theory of superconductivity[1] is often naively introduced as Bose-Einstein condensation (BEC) of bound pairs of electrons in the spin-singlet channel, which is represented by an antisymmetric spin wave-function of one spin-up electron and one spin-down electron. Breaking such a bound pair would cost twice of an energy gap, so it seems impossible for the two electrons to go separate ways without first obtaining this energy. If this naive picture were correct, it would be inconceivable for the condensate of a singlet-pairing superconductor (SPSC) to be able to carry a dc spin current, which is defined as the spin-up and -down electrons flowing in opposite directions (relative to some spin quantization axis), with no net charge flow. Fortunately the BCS state is much more subtle than that. Thus, as an important difference between a singlet-pairing BCS state and a BEC state of tightly bound, spin-singlet fermion pairs, we show below that in a conductor carrying a moderate spin current, spin-singlet Cooper pairing[2] of electrons can still take place almost as effectively as without the spin current. An energy gap still opens up on both the spin-up- and -down Fermi surfaces, which are now centered at $\pm\mathbf{q}/2$ momentum, and are no longer in overlap. [3] Transition temperature and energy gap are only extremely weakly reduced from those in the absence of a spin current, and all universality relations and laws of corresponding states found in the BCS theory should remain valid. Since the imposed spin current is found to co-exist with spin-singlet pairing at absolute zero temperature, when the system is fully gapped and no excitations can possibly exist, we conclude that a spin current can flow persistently and without dissipation in a SPSC. The spin current should thus be called a super-spin-current (SSpC), and the system, a super-spin-current-conductor (SSpCC). Two experimental tests of this prediction are proposed below.

First, we offer a heuristic picture: Consider n dancing couples arranged in a big circle with each couple circling a small circle at diametrically opposite positions like a bound state of two electrons. If each couple is also advancing along the big circle, it would be the analog of a super(-charge)-current (SCHC). If all dancers now agree

that, in every five minutes, say, changing partners *coherently* should take place, with each man stepping forward and each woman stepping backward, along the big circle, until new dancing partners can be formed, one then finds that a new kind of flow is occurring in the floor, with all men advancing in the counterclockwise direction, and all women in the clockwise direction. This is an analog of a SpC, with a man (woman) the analog of a spin-up (-down) electron. If the intra-pair distance were smaller than half of the inter-pair distance, the pairs would have to be broken before new pairs can be formed. In the BCS state of a SC, however, the intra-pair distance — which is of the order of the coherence length [2] ξ — is much larger than the inter-pair distance, so that Cooper-pair orbits are in strong overlap, allowing this partner-changing process to take place without the need to break the pairs, and thus a SSpC can flow in the system. This partner-changing process would be drastically suppressed in a BEC state of tightly bound pairs, where the intra-pair distance is much smaller than the inter-pair distance, although quantum mechanical wave-functions do not cut off abruptly, so the probability that this process can happen is never zero. Thus we surmise that only an extremely weak SSpC can exist in such a BEC state, to the extend of being negligible.

For a simple theory, we begin with the second-quantized BCS Hamiltonian:

$$\mathcal{K}_0 \equiv \mathcal{H}_0 - \mu N = \sum_{\mathbf{k},s} \xi_{\mathbf{k}} \hat{c}_{\mathbf{k},s}^\dagger \hat{c}_{\mathbf{k},s} - (\lambda/2\Omega) \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}, s_1, s_2} \hat{c}_{\mathbf{k}_1+\mathbf{q}, s_1}^\dagger \hat{c}_{\mathbf{k}_2-\mathbf{q}, s_2}^\dagger \hat{c}_{\mathbf{k}_2, s_2} \hat{c}_{\mathbf{k}_1, s_1}, \quad (1)$$

where Ω the total volume of the system, $\lambda > 0$ an effective coupling constant, and $\xi_{\mathbf{k}} \equiv \hbar^2 \mathbf{k}^2 / 2m^* - \mu$, with m^* the electron effective mass, and μ the chemical potential. Adding to it the Zeeman Hamiltonian:

$$\mathcal{H}_1 = -h \sum_{\mathbf{k}, s, s'} \hat{c}_{\mathbf{k}, s}^\dagger \sigma_{ss'}^z \hat{c}_{\mathbf{k}, s'}, \quad (2)$$

(where σ^z is the third Pauli matrix, and h is the electron magnetic moment times an external magnetic field

in the z direction, and can be viewed as a Lagrange multiplier,) would favor a spin imbalance along z in the spin space. Similarly, to favor a ChC, one can introduce a vector Lagrange multiplier \mathbf{v}_{ch} , whose direction defines the direction of the charge flow, and add to \mathcal{H}_0 :

$$\mathcal{H}_2 = -\mathbf{v}_{ch} \cdot \sum_{\mathbf{k},s} \hbar \mathbf{k} \hat{c}_{\mathbf{k},s}^\dagger \hat{c}_{\mathbf{k},s} + (1/2) m \mathbf{v}_{ch}^2 \sum_{\mathbf{k},s} \hat{c}_{\mathbf{k},s}^\dagger \hat{c}_{\mathbf{k},s}, \quad (3)$$

where the second term is a correction to μ . Thus to favor a SpC in the system (with the same spin quantization direction), we introduce a vector Lagrange multiplier \mathbf{v}_{sp} , and add to \mathcal{H}_0 :

$$\mathcal{H}_3 = -\mathbf{v}_{sp} \cdot \sum_{\mathbf{k},s,s'} \hbar \mathbf{k} \hat{c}_{\mathbf{k},s}^\dagger \sigma_{ss'}^z \hat{c}_{\mathbf{k},s'} + (1/2) m \mathbf{v}_{sp}^2 \sum_{\mathbf{k},s} \hat{c}_{\mathbf{k},s}^\dagger \hat{c}_{\mathbf{k},s}. \quad (4)$$

For discussing a superconductor carrying no SChC, BCS neglected all interaction matrix elements that involves electron-pairs of non-zero momentum. Then a mean-field approximation changes \mathcal{K}_0 to:

$$\begin{aligned} \mathcal{K}_{0,MF} = & \sum_{\mathbf{k},s} \xi_{\mathbf{k}} \hat{c}_{\mathbf{k},s}^\dagger \hat{c}_{\mathbf{k},s} - \Delta^* \sum_{\mathbf{k}} \hat{c}_{-\mathbf{k},\downarrow} \hat{c}_{\mathbf{k},\uparrow} \\ & - \Delta \sum_{\mathbf{k}} \hat{c}_{\mathbf{k},\uparrow}^\dagger \hat{c}_{-\mathbf{k},\downarrow}^\dagger + (\Omega/\lambda) |\Delta|^2, \end{aligned} \quad (5)$$

$$\Delta \equiv (\lambda/\Omega) \sum_{\mathbf{k}} \langle\langle \hat{c}_{-\mathbf{k},\downarrow} \hat{c}_{\mathbf{k},\uparrow} \rangle\rangle_T, \quad (6)$$

with $\langle\langle \dots \rangle\rangle_T$ denoting grand-canonical ensemble average at temperature T . The Hamiltonian $\mathcal{K}_{3,MF} \equiv \mathcal{K}_{0,MF} + \mathcal{H}_3$ can be diagonalized in the same way as $\mathcal{K}_{0,MF}$: by using a Bogoliubov-Valatin transformation:

$$\hat{c}_{\mathbf{k},\uparrow} = u_{\mathbf{q},\mathbf{k}}^* \hat{\gamma}_{\mathbf{q},\mathbf{k},\uparrow} + v_{\mathbf{q},\mathbf{k}} \hat{\gamma}_{\mathbf{q},-\mathbf{k},\downarrow}^\dagger \quad (7)$$

$$\hat{c}_{-\mathbf{k},\downarrow}^\dagger = -v_{\mathbf{q},\mathbf{k}}^* \hat{\gamma}_{\mathbf{q},\mathbf{k},\uparrow} + u_{\mathbf{q},\mathbf{k}} \hat{\gamma}_{\mathbf{q},-\mathbf{k},\downarrow}^\dagger, \quad (8)$$

where $\mathbf{q} \equiv 2m^* \mathbf{v}_{sp}/\hbar$. The result is:

$$\begin{aligned} \mathcal{K}_{3,MF} = & \sum_{\mathbf{k}} E_{\mathbf{q},\mathbf{k}} (\hat{\gamma}_{\mathbf{q},\mathbf{k},\uparrow}^\dagger \hat{\gamma}_{\mathbf{q},\mathbf{k},\uparrow} + \hat{\gamma}_{\mathbf{q},-\mathbf{k},\downarrow}^\dagger \hat{\gamma}_{\mathbf{q},-\mathbf{k},\downarrow}) \\ & + \sum_{\mathbf{k}} (\xi_{\mathbf{q},\mathbf{k}} - E_{\mathbf{q},\mathbf{k}}) + (\Omega/\lambda) |\Delta_{\mathbf{q}}|^2, \end{aligned} \quad (9)$$

$$u_{\mathbf{q},\mathbf{k}} = \left[\frac{1}{2} \left(1 + \frac{\xi_{\mathbf{q},\mathbf{k}}}{E_{\mathbf{q},\mathbf{k}}} \right) \right]^{\frac{1}{2}} \quad (10)$$

$$v_{\mathbf{q},\mathbf{k}} = \frac{\Delta_{\mathbf{q}}}{|\Delta_{\mathbf{q}}|} \left[\frac{1}{2} \left(1 - \frac{\xi_{\mathbf{q},\mathbf{k}}}{E_{\mathbf{q},\mathbf{k}}} \right) \right]^{\frac{1}{2}}, \quad (11)$$

$$\xi_{\mathbf{q},\mathbf{k}} \equiv \xi_{(\mathbf{k}-\mathbf{q}/2)}, \quad (12)$$

$$E_{\mathbf{q},\mathbf{k}} = \sqrt{\xi_{\mathbf{q},\mathbf{k}}^2 + |\Delta_{\mathbf{q}}|^2}, \quad (13)$$

where $\Delta_{\mathbf{q}}$ is defined similar to Δ of Eq. 6. That $E_{\mathbf{q},\mathbf{k}}$ is positive definite for all \mathbf{q} shows that a SpC does not induce pair breaking in the system.

Many physical quantities can now be evaluated as in the original BCS theory: The difference between the superconducting and normal ground-state energies at $T = 0$ is:

$$\begin{aligned} \Delta E_3|_{T=0} & \equiv \langle\langle \mathcal{K}_{3,MF}^{(S)} \rangle\rangle_{T=0} - \langle\langle \mathcal{K}_{3,MF}^{(N)} \rangle\rangle_{T=0} \\ & = -(1/2) \mathcal{N}(0) \Omega |\Delta_{\mathbf{q},0}|^2, \end{aligned} \quad (14)$$

where $\mathcal{N}(0)$ is the density of states per spin at the $\mathbf{q} = 0$ Fermi energy, and the usual weak-coupling approximation has been assumed; $\Delta_{\mathbf{q},0} \equiv \Delta_{\mathbf{q}}|_{T=0}$. Let the cut-off energies be still $\pm \hbar \omega_c$ for energy integration around the ($\mathbf{q} = 0$) Fermi energy, the zero-temperature gap equation

$$1 = (1/2) \mathcal{N}(0) \lambda \int_{-1}^1 \frac{d\mu}{2} \left[\ln \frac{2(\hbar \omega_c - \hbar v_F q \mu)}{\Delta_{\mathbf{q},0}} + \ln \frac{2(\hbar \omega_c + \hbar v_F q \mu)}{\Delta_{\mathbf{q},0}} \right] \quad (15)$$

gives:

$$\Delta_{\mathbf{q},0} \simeq \Delta_{0,0} \exp \left[-\frac{1}{6} \left(\frac{\hbar v_F q / 2}{\hbar \omega_c} \right)^2 \right], \quad (16)$$

with $\Delta_{0,0}$ the $T = 0$ gap in the BCS theory. The gap equation at $T \neq 0$:

$$1 = \frac{\lambda}{\Omega} \sum_{\mathbf{k}} \frac{1}{2E_{\mathbf{q},\mathbf{k}}} \tanh \left(\frac{E_{\mathbf{q},\mathbf{k}}}{2k_B T} \right) \quad (17)$$

implies

$$T_{c,\mathbf{q}} = T_{c,0} \exp \left[-\frac{1}{6} \left(\frac{\hbar v_F q / 2}{\hbar \omega_c} \right)^2 \right], \quad (18)$$

where $T_{c,0}$ is the transition temperature in the BCS theory. Thus the universality relation of the BCS theory is obtained:

$$\frac{2\Delta_{\mathbf{q},0}}{k_B T_{c,\mathbf{q}}} = 2\pi e^C = 3.528, \quad (19)$$

with $C = 0.577$ the Euler constant. From Eq. 17, one can derive the same BCS universality relation between $|\Delta_{\mathbf{q}}/\Delta_{\mathbf{q},0}|$ and $T/T_{c,\mathbf{q}}$. The fractional specific-heat jump at $T_{c,\mathbf{q}}$, or $\Delta C/C_n|_{T_{c,\mathbf{q}}}$, is also found to be that of the BCS theory. Other thermodynamic quantities have not been investigated, but we expect all universality relations and laws of corresponding states in the BCS theory to remain valid. Evaluating the ensemble average of the SpC density:

$$\begin{aligned} \langle\langle \mathbf{j}_{sp} \rangle\rangle_T & \equiv (1/\Omega) \langle\langle \sum_{\mathbf{k}} (\hbar \mathbf{k} / m^*) (\hat{c}_{\mathbf{k},\uparrow}^\dagger \hat{c}_{\mathbf{k},\uparrow} + \hat{c}_{-\mathbf{k},\downarrow}^\dagger \hat{c}_{-\mathbf{k},\downarrow}) \rangle\rangle_T \\ & = \frac{1}{\Omega} \sum_{\mathbf{k}} \frac{\hbar \mathbf{k}}{m^*} \left(1 - \frac{\xi_{\mathbf{q},\mathbf{k}}}{E_{\mathbf{q},\mathbf{k}}} \tanh \frac{E_{\mathbf{q},\mathbf{k}}}{2k_B T} \right), \end{aligned} \quad (20)$$

which, in the weak coupling approximation, is always equal to its normal-state value $n\mathbf{v}_{sp}$ at all $T < T_{c,q}$ (and above), where n is the electron density, and \mathbf{v}_{sp} is the spin-current velocity. This finite SpC must be all SSpc at $T = 0$, because at $T = 0$ there is no normal fluid component and the system is fully gapped, and it must be all normal SpC at $T = T_{c,q}$, because we have a continuous phase transition to the normal state at this temperature. At any T between 0 and $T_{c,q}$ it must be partly super and partly normal, but we have not yet split apart the two components. The important point established here is that a SSpc exists and that a SPSC can carry a SSpc. If the corresponding ChC problem $\mathcal{K}_{0,MF} + \mathcal{H}_2$ is solved by the same approach, and the ensemble-average of the ChC density is calculated, one would obtain

$$\langle \mathbf{j} \rangle_T = \frac{2e\hbar}{m} \frac{1}{\Omega} \sum_{\mathbf{k}} \mathbf{k} f\left(-\frac{\hbar\mathbf{k}}{m^*} \cdot \frac{\mathbf{q}}{2} + E_{\mathbf{k}}\right), \quad (21)$$

($\mathbf{q} \equiv 2m^*\mathbf{v}_{ch}/\hbar$) which vanishes at $T = 0$ and is equal to the normal-state value $ne\mathbf{v}_{ch}$ at $T = T_c$. This ChC is all normal at any T : The Hamiltonian $\mathcal{K}_{0,MF} + \mathcal{H}_2$ is just $\mathcal{K}_{0,MF}$ in the frame moving with the velocity \mathbf{v}_{ch} . In that frame the normal fluid component is stationary, whereas the pair condensate is moving with velocity $-\mathbf{v}_{ch}$. Thus $\langle \mathbf{j} \rangle_T$ in that frame is all SChC. Translating back to the lab frame, we obtain only normal current, since the pair condensate is now stationary. Such a trick of going to a moving frame can not be played to the SpC problem. Our mean-field solution for $\mathcal{K}_{0,MF} + \mathcal{H}_3$ is also not a mean-field solution of $\mathcal{K}_{0,MF}$ alone, which might bother some readers, as it seems that the “fictitious field” \mathbf{v}_{sp} cannot be realized in the laboratory. [4] Our reply is: We think that a superconducting sample subject to an external SpC (in the x direction, say) can be modeled as the Hamiltonian \mathcal{K}_0 together with the boundary condition $\hat{\psi}_\sigma(\mathbf{x} + L\hat{e}_x) = e^{-i(q/2)L\sigma} \hat{\psi}_\sigma(\mathbf{x})$ for the field operator $\hat{\psi}_\sigma(\mathbf{x})$. If one then makes a gauge transformation $\hat{\psi}_\sigma(\mathbf{x}) = e^{-i(q/2)x\sigma} \hat{\psi}'_\sigma(\mathbf{x})$, one would obtain $\mathcal{K}_0 + \mathcal{H}_3$ with \mathbf{v}_{sp} along x and the usual periodic boundary conditions, which we have solved. This gauge transformation changes the eigen-wave-functions, but not the eigen-energies, nor $\Delta_{\mathbf{q}}$.

We expect v_{sp} to be comparable in magnitude to v_{ch} , and is of the order of 1 mm/s or less. Thus $\hbar v_F(q/2)/\hbar\omega_c = m^*v_F v_{sp}/\hbar\omega_c$ appearing in Eqs. 16 and 18 is of the order of 10^{-7} or smaller. Thus $\Delta_{\mathbf{q},0}/\Delta_{0,0}$ and $T_{c,q}/T_{c,0}$ are exceedingly close to unity. We conclude that all thermodynamic properties of a superconductor are practically not affected by the presence of a laboratory-realizable SpC in the system.

Increasing the magnitude of a SChC velocity can lead to two critical values: At the first, Landau critical velocity, depairing begins, and at the second one the order parameter is suppressed to zero [5] (in three dimensions — In < 3 dimensions the two critical velocities merge into

one [6]). No depairing can be induced by a SSpc velocity as long as weak-coupling approximation is valid. So no critical SSpc velocity can be defined this way. Still, a critical SSpc velocity likely exists for $\hbar v_F q > \hbar\omega_c$, but its value would be so large that it is practically irrelevant. The Landau critical velocity is already quite large, but for SChC, there exist lower critical velocities due to the creation of phase-slip centers in one dimension and super-current vortices (or flux lines) in higher dimensions. The decay mechanisms for SSpc carried by a SPSC remain to be investigated,[7] but it appears that a SPSC can at least carry comparable SSpc as SChC (after artificially introducing a factor of $|e|$ to the definition in Eq. 20 so they can be compared). This is potentially useful to the development of SpC circuits and for spin injection, as it appears that SpC can not flow in most normal conductors for macroscopic distances.

If the length L is that of the circumference of a ring sample, then $(q/2)L = 2n\pi$ for an integer n , so that $\hat{\psi}_\sigma(\mathbf{x})$ can satisfy the usual periodic boundary conditions. The SSpc is then quantized as a SChC in such a situation. Thus line singularities in SSpc can also exist, as vortices (flux lines) in SChC, even as phase gradient of the pair wave function is no longer a relevant concept to SSpc.

The ultimate confirmation of this prediction must be via experimentation. We thus propose the following two experimental tests of our prediction:

In the first proposed test, one should simply connect a SPSC to a battery except that at least one section of the connection wiring should be made of a half-metallic conductor (HMC) [8], allowing, say, only spin-up current. If the SPSC can carry a dc SSpc (referred to as “scenario A”), then it can also carry a dc super-spin-up-current, since an equal mixing of a SpC and a ChC is just a pure spin-up current. No voltage drop will occur across the SC, so the whole voltage drop will exist in the leads, driving the same amount of pure spin-up current in the leads. On the other hand, if the SPSC can not carry a dc SSpc (referred to as “scenario B”), then spin-up current cannot flow through the SC, and will initially generate spin-up charge accumulations at the interfaces between the leads and the SC. Spin-neutral SChC will then be induced in the SC, until the spin-up charge accumulations at the said interfaces are all converted to pure spin-density accumulations. The SChC will then stop. The “SpC voltage” [9] established in the leads will then cancel out the effect of the battery voltage on the spin-up charges in the lead, (but they will add up on spin-down charges which, however, can not flow in the leads,) so that no net electro-chemical-potential gradient will be acting on the spin-up electrons in the leads, and no net voltage will be acting across the SC. The final state is a zero current state. The difference between these two scenarios can be easily differentiated by a current meter in the circuit, or by the magnetic field generated by the

spin-up current in the scenario A only. If the HMCs are only nearly perfect, then some small ChC will always flow in the circuit, which becomes SChC in the SC, but the actual total current flowing in the circuit will be much larger in scenario A than in scenario B. In scenario A the proper ratio of spin-up and -down currents will flow in the circuit, as if the SC does not exist in the circuit, whereas in scenario B, the final state is such that only a small amount of spin-neutral ChC will flow in the circuit, which becomes pure SChC in the SC. Spin-density accumulations will exist at the said interfaces, to reduce the electro-chemical-potential gradient of the spin-up electrons, and enhance that of the spin-down electrons, by so much so that spin-up and -down current densities in the circuit can be equal. This is a self-consistency problem which will be investigated in the future.

In the second proposed test, a SC strip is shaped into a ring with a narrow gap, which is filled with a HMC allowing only spin-down current, with good contacts with both ends of the SC strip to form a closed circuit of low resistance. A battery with two HMC leads connected to the two ends of the SC strip are used to send a pure spin-up current through the SC strip. In scenario B no ChC or SpC will flow through the SC. In scenario A the battery will succeed in sending a spin-up current through the SC strip, but to reduce the magnetic energy associated with the magnetic field induced in the loop, we think that a spin-down current will be induced in the ring, so that in the SC there is only a SSpC but no SChC. To distinguish between these two scenarios one can use either a current meter to detect the spin-up current in scenario A only, or a thermometer placed in contact with the HMC filling the gap of the ring in order to detect the heat generated by the spin-down current in it in scenario A only. In either scenarios there is little magnetic field in the ring to be detected. Imperfect HMCs can be similarly analyzed.

Hirsch has pointed out [10] that if the BCS wave function is altered to break parity but not the time-reversal symmetry, it can then carry a dissipation-less SSpC. Thus the ground-state wave-function obtained here as a SpC-carrying wave-function is not new, but as the mean-field ground-state wave-function of a meaningful Hamiltonian associated with a SpC in the system it is new. In addition, the main purpose of Ref. [10] is to discuss when a symmetry-breaking SCing (i.e., pairing) condensate will carry also a spontaneous SpC. Thus it has to find special model Hamiltonians to achieve this goal. On the other hand, the main purpose of the present work is to discuss what happens if one attempts to send a pure SpC through an ordinary, SPSC. We conclude that *all* SPSCs can let a SpC flow through without dissipation, and the SCing properties of such a system is little affected by any moderate SpC flowing in it. Furthermore, a quantized, persistent, dissipation-less SSpC can exist in any ring sample of any ordinary SPSC. No special Hamiltonian is needed to achieve either goals. We have also proposed

two currently feasible experimental tests of this prediction. Spin-orbit interaction has not yet been considered in this work. This interaction led Hirsch [11] to propose a spin Meissner effect as a SC response to ionic electric fields.

The concept of SSpCC (or SpC SC) has also been introduced in a very different context in spin-orbit-split hole-bands of semiconductors such as Si or GaAs, without invoking pairing at all. [12, 13] An *off-diagonal* transport coefficient was considered there, which is naturally dissipation-less. But when diagonal transport coefficients are considered at the same time, the SpC may no longer be dissipation-less. The SSpC introduced in the present work is associated with a diagonal transport coefficient — SpC conductivity: If a dc SpC can exist persistently and without dissipation, this diagonal transport coefficient must diverge in the zero-frequency limit, just like the dc (ChC-)conductivity in a SC. Thus the concept of SSpCC introduced here is genuine. Its potential for practical applications should be enormous.

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* Electronic address: crhu@tamu.edu

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